Q10 4.2-4.4

1) Given the curve y = x^2 over the interval [1,3],

(a) Find the exact value for the area under the curve using the Riemann sums definition of an integral using right endpoints of each subinterval as sample points. Show details carefully. (see example 3 in book or video 4.2 @ 22:22))

(b) Find the area exactly using the FTC part 2 and the antiderivative. (i.e. integrate directly)

(11 points)

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

$$\boxed{\Delta x = \frac{b-a}{n}} = \frac{3-1}{n} = \frac{2}{n}$$

$$\boxed{x_i = a + i\Delta x} = 1 + i\frac{2}{n}$$

$$\boxed{f(x_i)} = x_i^2 = \left(1 + i\frac{2}{n}\right)^2 = 1 + \frac{4}{n}i + \frac{4}{n^2}i^2$$

a) $$A = \int_1^3 x^2\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(1 + \frac{4}{n}i + \frac{4}{n^2}i^2\right)\frac{2}{n}$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[\sum_{i=1}^{n} 1 + \frac{4}{n}\sum_{i=1}^{n} i + \frac{4}{n^2}\sum_{i=1}^{n} i^2\right]$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[n + \frac{4}{n}\frac{n(n+1)}{2} + \frac{4}{n^2}\frac{n(n+1)(2n+1)}{6}\right]$$

$$= \lim_{n\to\infty}\left[2 + \frac{4}{n^2}n(n+1) + \frac{4}{3n^3}n(n+1)(2n+1)\right]$$

$$= \lim_{n\to\infty}\left[2 + \frac{4}{n}(n+1) + \frac{4}{3n^2}(n+1)(2n+1)\right]$$

$$= 2 + 4 + \frac{8}{3} = \boxed{\frac{26}{3}}$$

b) $$\int_1^3 x^2\,dx = \frac{x^3}{3}\Big]_1^3 = \frac{27}{3} - \frac{1}{3} = \boxed{\frac{26}{3}}$$

3) Evaluate the following integrals (3 points each)

a) $\int_0^{\pi/4}\left(\cos x+\sec^2 x\right)dx = \sin x + \tan x\Big]_0^{\pi/4} = \sin\frac{\pi}{4}+\tan\frac{\pi}{4} - (\sin 0 + \tan 0)$

$= \boxed{\frac{\sqrt{2}}{2}+1}$

b) $\int_1^8\left(x^{2/3}-4x\right)dx = \frac{3}{5}x^{5/3} - 2x^2\Big]_1^8$

$= \frac{3}{5}\cdot 8^{5/2} - 128 - \left(\frac{3}{5}-2\right)$

$= \frac{3}{5}\cdot 32 - 128 - \frac{3}{5} + 2$

$= \frac{96}{5} - \frac{3}{5} - 126 = \frac{93}{5} - 126 = \boxed{\frac{-537}{5}}$

c). $\int v^2(v^4+5)\,dv = \int(v^6+5v^2)dv$

$= \boxed{\frac{1}{7}v^7 + \frac{5}{3}v^3 + C}$